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The homomorphisms between the Dickson-Mui algebras as modules over the Steenrod algebra. (English summary)

The Dickson-Mui algebra $D(V)$ of an elementary abelian $p$-group $V$ consists of all invariants in the mod $p$ cohomology $H^*(V)$ with respect to the action of the general linear group $GL(V)$. It is a module over the Steenrod algebra $A$. As an $A$-module $D(V)$ splits as the direct sum of its augmentation ideal $\overline{D}(V)$ (the reduced Dickson-Mui algebra of $V$) and $F_p$, given by the unit of $D(V)$. The author determines an explicit basis of the vector space of all $A$-module homomorphisms between $\overline{D}(V)$ and $\overline{D}(W)$. The basis elements are given as compositions of transfers and restrictions passing through intermediate reduced Dickson-Mui algebras $D(U)$, where $U$ is an elementary abelian $p$-group with $\text{rank}(U) \leq \min\{\text{rank}(V), \text{rank}(W)\}$.

Furthermore, for $V = W$ the author determines the algebra of all $A$-module endomorphisms of $D(V)$ (resp. $\overline{D}(V)$); it is isomorphic to an explicit quotient of a polynomial algebra on one indeterminate. As a consequence it is shown that the reduced Dickson-Mui algebras are atomic in the sense that if an $A$-module endomorphism of the algebra is nonzero on the least positive degree generator, then it is an automorphism. This implies in particular that the reduced Dickson-Mui algebra is an indecomposable $A$-module. Similar results also hold for the odd characteristic usual Dickson algebras given as the invariants of the action of $GL(V)$ on the symmetric algebra of the dual of $V$, again considered as an $A$-module.

In the case of the algebra of automorphisms of $D(V)$ (resp. $\overline{D}(V)$) similar results have been found in an unpublished paper of N. E. Kechagias ["A Steenrod-Milnor action ordering on Dickson invariants", submitted, see http://www.math.uoi.gr/~nondas_k/Steen-Mil.pdf].

The author uses as a major tool the injectivity of $H^*V$ as an object in the category $\mathcal{U}$ of unstable $A$-modules, as well as the Adams-Gunawardena-Miller Theorem, which states that the canonical homomorphism $F_p[\text{Hom}(V,W)] \to \text{Hom}_\mathcal{U}(H^*W, H^*V)$ is an isomorphism. Using these tools a bit more systematically leads to a positive answer to Conjecture 1.5 of the paper. In fact, injectivity of $H^*V$ implies that for any subgroups $G \subset GL(V)$ and $H \subset GL(W)$ there are canonical isomorphisms


(the latter object denoting the $G$-invariants of the $H$-coinvariants!), and then the Adams-Gunawardena-Miller Theorem gives

$$\text{Hom}_\mathcal{U}((H^*W)^H, (H^*V)^G) \cong (F_p[\text{Hom}(V,W)]_H)^G \cong F_p[H/\text{Hom}(V,W)/G],$$

i.e. precisely Conjecture 1.5. If $V = W$ and $H = G$ then composition on the left-hand side corresponds to the product in the Hecke algebra of double cosets on the right-hand side, thus giving an answer to another question posed by the author in the context of Conjecture 1.5.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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