An improved numerical scheme for the generalized Burgers–Fisher equation

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A fourth order in time modified predictor-corrector method is proposed for the numerical solution of the generalized Burgers–Fisher (BgF) equation, which is given by

$$u_t + \alpha u^{\delta}\, u_x - u_{xx} = \beta u \left(1 - u^{\delta}\right)\ ;\ \ 0 \le x \le 1\,,\ \ t > 0 \tag{1}$$

with initial condition $u\,(x, 0) = f(x)$; $x \in [0, 1]$ and boundary conditions $u_x|_{x=0,\,1} = 0$; $t > 0$. The real-valued function $u = u\,(x, t)$ is a sufficiently often differentiable function of the space and the time variables respectively; α, β are real parameters and δ is a positive integer.

Eq. (1) has a wide range of applications in plasma physics, fluid physics, capillary-gravity waves, nonlinear optics and chemical physics, while many researchers have studied it theoretically and numerically (see, e.g., [1, 2] and references therein).

The main aim of this paper is to solve the BgF equation explicitly with a direct method. To this attempt, the solution of the resulting nonlinear system is given by expressing the unknown vector component wise and updating each component as soon as its value becomes available. This process, which is known as a modified predictor-corrector method (see, e.g., [3] and references therein), opposite to the iterative classical predictor-corrector one is always explicit and is applied once, has also been examined successfully with various other approximations in time giving an improvement in the accuracy over the classical method.

References

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