On a way for the numerical solution of Volterra integro-differential equations

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In the chronology of scientific papers is noticeable that scientists study the solution of integro-differential equations involved later than the study of differential and integral equations. Consequently, the fundamental studied are the differential equations and, therefore, many experts are trying to replace the solving of integral and integro-differential equations to solving of differential equations. Extending this idea, constructed here a general hybrid method for solving integro-differential equations, and investigates the equivalence of the above-mentioned equations. Constructed here the concrete stable hybrid method with high accuracy by using mesh points in a minimal amount, but rather $k = 1$ constructed the method of the degree $p = 6$.

Introduction. It is known that many applied and scientific problems can be reduced to the solution of the integro-differential equation, which in one variant, can be written in the following form:

$$y'(x) = f(x, y) + \int_{x_0}^{x} K(x, s, y(s))ds, x_0 \leq s \leq x \leq X. \tag{1}$$

This equation is usually called as Volterra integro-differential equation by which many scientists have studied the problem of environmental geophysics, etc. As it is known, in the middle of the twentieth century, scientists have investigated hybrid methods for the construction of numerical methods with improved properties. The use of multi-step hybrid methods for the numerical solution of differential equations has become more popular after the publication of the famous work Geare and Butcher. And one of the first papers on the using of hybrid methods for solving integral and integro-differential equations, is the work Makroglou.

Here, for the numerical solving equation (1) with initial conditions $y(x_0) = y_0$ used the following finite-difference method with the constant coefficients:

$$\sum_{i=0}^{k} \alpha_i z_{n+i} = \sum_{i=0}^{k} \beta_i z'_{n+i} + h \sum_{j=0}^{k} \gamma_i z'_{n+i+\nu_i}, (|\nu_i| < 1, i = 0, 1, ..., k). \tag{1}$$

To compare the methods to be constructed here with known methods, the solution of equation (1), is reduced to solving a system consisting only of differential or integral equations, and also considered replacing the solution of equation (1) to the solution of equations with the differential and integral equations. A comparison of the results obtained by known methods and methods suggested here.