Efficient modified Newton's methods for solving systems of nonlinear equations

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The problem of solving systems of nonlinear equations is of great significance in the fields of computational science and engineering. Newton's method is the most well known method for solving such problems, with quadratic convergence to be ensured, whenever a sufficiently good initial approximation and a nonsingular Jacobian matrix are available. These situations may restrict its application and thus several interesting modifications of Newton's method have been proposed to contribute in this active area of research.

In this work, we propose two modifications of Newton's method mainly to cope with the above difficulties. The first proposed method may be considered as belonging to the class of quasi Newton methods, based on updating the Jacobian matrix through new defined points. The second method is an improvement of the first one for managing problems with imprecise function values. For both methods, it is proved that they retain the important Newton's advantage to converge quadratically. Under some assumptions, the enlargement of convergence area may be succeeded.

The first preliminary numerical results show the superiority of proposed methods in terms of Newton's one, regarding the number of iterations, the total computational cost and the dependency of initial points. In case of singular Jacobian matrix, their efficiency is also tested successfully.