A relaxation lexicographic optimization method for the unconstrained optimization problem

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The mathematical problem for the unconstrained optimization problem is:

$$\min_x f(x),$$

where $x \in \mathbb{R}^n$ is a real vector with $n \geq 1$ and $f : \mathbb{R}^n \to \mathbb{R}$ is a smooth function.

In Lexicographic Optimization approach a sequence of different optimization problems is solved according to a given order of priority. In a recent work, we have incorporated with the Lexicographic Optimization idea for unconstrained optimization problem, as a way to reach the area of the minimum points from different directions, with promising results. Utilizing this idea, simplified objective functions are created for the intermediate optimization problems and it worths noting that they are not optimized exhaustively. By this way, a better initial point is gradually achieved for the last problem of the sequence of problems, which essentially corresponds to the original unconstrained optimization one.

Taking into account the experience gained from the above implementation, an improvement of this approach is introduced in the present paper. Specifically, relaxation parameters on an appropriate partition of the original objective function are used to create better intermediate objective functions. The convergence of the proposed method is proved. Additionally, it is applied on a set of test functions and the numerical results are rather promising.