Dynamics of fractional-order Hanta epidemic model using a nonstandard numerical scheme

Mevlüde Yakıt Ongun [a], Roberto Garrappa[b]

[a]Department of Mathematics, Suleyman Demirel University,

Isparta, Turkey

[b]Department of Mathematics, University of Bari,

Bari, Italy

mevludeyakit@sdu.edu.tr,garrappa@dm.uniba.it

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Although fractional calculus has a long history, its applications to natural science are just a recent focus of interest. The description of some phenomena is more accurate when the fractional derivative is used.

In this paper, we introduce the fractional order into the model of Hanta virus epidemics in Abramson (2002). The new system is given as the set of fractional differential equations (FDEs):

$$\left.\begin{array}{c} {}^{C}D^{\alpha}_{t_0}X(t) = (b-c)X(t) + bY(t) - \frac{X(t)^2}{K} - \left(\frac{1+aK}{K}\right)X(t)Y(t) \\ {}^{C}D^{\alpha}_{t_0}Y(t) = -cY(t) - \frac{Y(t)^2}{K} - \left(\frac{1-aK}{K}\right)X(t)Y(t) \end{array}\right\} \tag{1}$$

$$X(t_0) = X_0, Y(t_0) = Y_0 \tag{2}$$

In this model, the fractional derivatives are used to describe nonhomogeneous character of the ecosystems, with respect to the presence of competitors. The parameter α is related to the density of competitor species in the system. When the order of the derivative operator $\alpha = 1$, this corresponds to integer order differential equation, and the competitor's populations varies when $0 < \alpha < 1$. As $\alpha > 1$, the density of competitor or alien species will increase in the populations.

We show that this equation system has a unique solution with given initial conditions on $t \succeq 0$. A nonstandard finite difference scheme is implemented to study the dynamics behaviours of fractional order Hanta epidemic system. We show that the method preserves the positivity properties of the integer order system. Some numerical solutions are illustrated by means of some graphs.