Spectral and structural analysis of FD matrix sequences arising in semi-elliptic differential equations

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We consider the 2D problem

$$-\frac{\partial}{\partial x}\left(a(x,y)\frac{\partial}{\partial x}u(x,y)\right)-\frac{\partial}{\partial y}\left(b(x,y)\frac{\partial}{\partial y}u(x,y)\right)=f(x,y) \tag{1}$$

with Dirichlet boundary conditions. Using the well-known five points formula and by ordering the unknowns in the classic manner, we arrive to the $n^2 \times n^2$ linear system

$$A_{nn}x = b,$$

For designing multigrid methods or effective preconditioners we have to allocate the space and the source of ill conditioning. The main contribution of this talk is to clarify the picture of the spectral conditioning of A_{nn}, under the assumptions of $a(x,y), b(x,y) \geq 0$ having distinct roots. Specifically, we will give formulas concerning the relations between the orders of the zeros of $a(x,y), b(x,y)$ and the asymptotic behavior of the minimal eigenvalue of the related matrices. We will show that there are two sources of ill conditioning, one comes from the Laplace operator and the other from the roots of $a(x,y)$ and $b(x,y)$ and these spaces, in general, do not interfere.