An hp Finite Element Method for fourth order singularly perturbed problems

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We consider the following fourth order singularly perturbed boundary value problem (BVP): Find $u(x)$ such that

$$\varepsilon^2 u^{(4)}(x) - \alpha(x)u''(x) + \beta(x)u(x) = f(x) \text{ in } I = (a,b)\,, \tag{1}$$

$$u(a) = u'(a) = u(b) = u'(b) = 0, \tag{2}$$

where $\alpha \geq 0, \beta \geq 0$ and f are given (sufficiently smooth) functions and $\varepsilon \in (0,1]$ is a given parameter (that can approach 0). In general, the solution to (1), (2) will contain *boundary layers* as $\varepsilon \to 0$; these are rapidly varying solution components that have support in a narrow region near the boundary.

We are interested in the approximation of the solution to (1), (2) by the hp version of the Finite Element Method (FEM). In particular, we construct suitable C^1 basis functions that allow us to show that the hp version of the FEM on the so-called *Spectral Boundary Layer Mesh* can give a robust approximation that converges exponentially, provided the data of the problem is analytic. The convergence is independent of ε and does not deteriorate as $\varepsilon \to 0$. In addition to presenting the method and its analysis, we will also show the results of numerical computations that validate the theory.