Error Control and Adaptivity for Advection-Dominated Problems

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Advection-dominated advection-diffusion-reaction elliptic and parabolic problems are known to be challenging numerically, due to the stiffness created by the interplay between the advection and the diffusion. In the context of finite element methods, a number of techniques which enhance the quality of the numerical approximations have been proposed over the last 25 years or so. Roughly speaking, such techniques are generally based on the careful introduction of numerical diffusion to reduce any numerical oscillations, created due to the stiffness of the problem. Such stabilised methods, therefore, aim to provide qualitatively good approximation in regions where the solution is smoothly varying, e.g., away from boundary and/or interior layers that are commonly featured in the PDE solution of these problems. In regions where the solution is rapidly varying, the numerical approximations need to be sufficiently locally resolved for good accuracy. Therefore, it is of interest to automatically control the error of stable numerical methods via adaptive algorithms. In this talk, I will review some new results on the design and performance of adaptive algorithms based on various types of a posteriori error estimation for such problems. The problem of error control for pure advection problems will also be considered. The question of the mesh design, e.g., the use of anisotropic meshes or hp-version finite element spaces, which is closely related to the complexity considerations of these algorithms, will also be discussed.