# Time-discrete higher order ALE formulations: A posteriori error analysis

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ALE formulations are useful when approximating solutions of problems in deformable domains, such as fluid-structure interactions. For realistic simulations involving fluids in 3d, it is important that the ALE method is at least of second order of accuracy. Second order ALE methods in time, without any constraint on the time step, do not exist in the literature and the role of the so-called geometric conservation law (GCL) for stability and accuracy is not clear. Recently, we were able to propose unconditionally stable discontinuous Galerkin (dG) methods in time of any order, for a time-dependent advection-diffusion model problem in moving domains, by involving appropriate quadrature in time that generalizes the GCL. Optimal order a priori error bounds were also established. Moving a step forward, in this talk, we discuss a posteriori error analysis for the mentioned dG methods applied to problems on moving domains. Key ingredients for the analysis are the definition of an appropriate reconstruction and the extension of certain projections on time-dependent domains. The proposed reconstruction is a generalization of the dG time reconstruction, introduced earlier by Makridakis & Nochetto for the corresponding equations on time independent domains. Using p.d.e. techniques, as for the original problem written in the ALE framework, we manage to prove optimal order a posteriori error bounds. The a posteriori error control gives important information on the behavior of the error with respect to the movement of the domain. In particular, our analysis allows variable time steps and suggests that time adaptivity is essential for oscillatory ALE maps. Numerical experiments confirm and complement our theoretical results.

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