Workshop

on

CONTEMPORARY DIFFERENTIAL GEOMETRY

Ioannina, 24th-27th September 2014

Abstracts

Alikakos Nicholas (University of Athens)

On the structure of phase transition maps: Density estimates and applications.

The scalar Allen-Cahn (or Ginzburg-Landau) equation is related to Minimal Surfaces and Minimal Graphs via the level sets of its solutions. The 1979 De Giorgi conjecture for the scalar problem has been settled relatively recently in a series of papers by Ghoussoub and Gui (2d), Ambrosio and Cabre (3d) , Savin (up to 8th dimension) and Del Pino, Kowlaczyk and Wei (counterexample for 9 dim. and above). The vector Allen-Cahn is related to Plateau Complexes. These are non-orientable minimal objects with a hierarchical structure. After explaining these relationships we focus on vector extensions of the Caffarelli-Cordoba Density Estimates (L. Caffarelli and A. Cordoba, Comm. Pure and Applied Mathematics Volume 48, Issue 1, pages 1–12, January 1995). In particular we establish lower co-dimension density estimates. These are useful for studying the hierarchical structure of certain entire vector solutions. We also give applications to minimal solutions (lower bounds, Liouville theorems).

Androulidakis Iakovos (University of Athens)

The Geometry behind Analysis.

The idea that equations can be solved once we understand the symmetries involved goes back to Galois. This principle was exploited by Sophus Lie in the context of differential equations and later by Emmy Noether to find conservation laws of classical systems. In this talk we will discuss how this trail of thought provides answers to contemporary problems, such as the calculation of gaps in the spectrum of Schrödinger-type operators and the detection of metrics with positive scalar curvature using Dirac operators and index theory. The appropriate symmetries for such problems are the external ones, codified nicely by the notion of a groupoid instead of a group. Time permitting we will state some open problems.

Bourni Theodora (Freie Universität Berlin)

"Null mean curvature" flow and marginally outer trapped surfaces.

In this talk we discuss a new second order parabolic evolution equation for hypersurfaces in space-time initial data sets, that generalizes mean curvature flow (MCF). In particular, the "null mean curvature" - a space-time extrinsic curvature quantity - replaces the usual mean curvature in the evolution equation defining MCF. This flow is motivated by the study of black holes and mass/energy inequalities in general relativity. We present a theory of weak solutions using level-set methods and an appropriate variational principle, and outline a natural application of the flow as a parabolic approach to finding outermost marginally outer trapped surfaces (MOTS), which play the role of quasi-local black hole boundaries in general relativity. This is joint work with Kristen Moore.

Dajczer Marcos (Instituto de Matematica Pura e Aplicada)

Rigidity, deformability and uniqueness aspects of submanifolds.

I will present in a series of 2 lectures an introduction to submanifold geometry. In particular, the organization of the lectures will be as follows:

a) In the first lecture I will provide a quick introduction the the basic equations of a submanifold. In the sequel I will present the fundamental theorem of submanifold theory. Moreover, I will focus to rigidity and deformability aspect of submanifold theory.
b) In the second lecture I will discuss global rigidity and real Kähler submanifolds.

Dajczer Marcos (Instituto de Matematica Pura e Aplicada)

Entire bounded constant mean curvature Killing graphs.

We show that under certain curvature conditions of the ambient space an entire Killing graph of constant mean curvature lying inside a slab must be a totally geodesic slice.

Fotiadis Anestis (Aristotle University of Thessaloniki)

Harmonic extensions of quasi-symmetric maps.

In this talk we study harmonic maps between hyperbolic spaces. In particular, we focus on a famous conjecture due to Rick Schoen. According to this conjecture *every quasi-symmetric map* $\varphi : \mathbb{S}^1 \to \mathbb{S}^1$ *can be uniquely extended to a quasi-conformal harmonic diffeomorphism* $u : \mathbb{D}^2 \to \mathbb{D}^2$, where here $\mathbb{D}^2$ stands for the unit open disk of $\mathbb{R}^2$ equipped with the hyperbolic metric. The uniqueness has been settled by Li and Tam. However, the existence part remains still open. We shall confirm affirmatively the conjecture if we impose some natural additional conditions on the initial data. Some of the results that we will present are joint with Professor Michel Marias.

Petalidou Fani (Aristotle University of Thessaloniki)

On the symplectic realization of Poisson-Nijenhuis manifolds.

We consider the problem of the symplectic realization of a Poisson-Nijenhuis manifold in the sense of F. Magri and C. Morosi. By applying a new technique presented by M. Crainic and I. Marcut for the symplectization of a Poisson tensor field we arrive to establish our problem and to prove, under conditions, the existence of a non-degenerate Poisson-Nijenhuis structure on an open neighborhood of the zero-section of the cotangent bundle of the manifold.

Platis Ioannis (University of Crete)

Introduction to sub-Riemannian geometry.

I will present in a series of 3 lectures an introduction to sub-Riemannian geometry. In particular, I will focus in the following topics:

a) Models of sub-Riemannian geometry: Examples from Physics, Heisenberg group, Isoperimetric problems.

b) General theory: Carnot-Caratheodori metric, Chow's theorem and existence of geodesics, geodesics and spheres in Heisenberg group.

c) Carnot groups, nilpotent discrete groups.

Savas-Halilaj Andreas (AUTH and LUH)

Translating solitons of the mean curvature flow.

In this talk I will present uniqueness and topological results concerning translating solitons of the mean curvature flow. More precisely, using Alexandrov's reflection principle I will show that a complete embedded translator of the mean curvature flow with a single end that is asymptotic to a paraboloid must coincide with the translating paraboloid. Moreover, I will show how the Gauss image affects on the topology of the translators. I will also mention some conjectures and open problems concerning the geometry of translators. This a joint work with Francisco Martin (Granada) and Knut Smoczyk (Hannover).

Smoczyk Knut (Leibniz Universität Hannover)

Mean curvature flow in higher codimension.

I will explain the concept of mean curvature flow of submanifolds in Riemannian manifolds of arbitrary codimension. This will be an introduction to the non-experts who want to learn methods in geometric evolution equations, e.g. the maximum principle and geometric comparison theorems. I will then focus on higher codimension, in particular on Lagrangian mean curvature flow and on the graphical mean curvature flow.

Sourdis Christos (University of Crete)

One-dimensional symmetry and related rigidity results for entire solutions to elliptic equations of Allen-Cahn type and their counterparts from minimal surface theory.

It is well known that a class of entire solutions to elliptic equations of Allen-Cahn type are related to minimal surfaces through their (unstable) level sets. The problem of showing one-dimensional symmetry of entire solutions to such equations has received considerable attention in the last years, motivated mainly by De Giorgi's conjecture (the analog of Bernstein's problem for minimal graphs) as well as that of Gibbons. Both conjectures have essentially been settled by now. In the first part of the talk, we will review these results.

Gibbons conjecture implies that if an (unstable) level set of a solution is bounded from above and below by two parallel hyperplanes, then that solution is one-dimensional (and that level set is a hyperplane). In the second part of this talk, we will show that the latter assertion still holds if a level set is bounded only from above (or below) by a hyperplane provided that it touches it at some point. In particular, our proof makes use for probably the first time of an old one-dimensional symmetry result of Caffarelli, Garofalo and Segala. We will also suggest a connection of our result to the strong maximum principle from the theory of minimal surfaces.

Our approach is based on Modica's gradient bound and on qualitative properties of solutions to autonomous semilinear elliptic equations, such as the famous sliding method, which we will recall. Based on this approach and the Hamiltonian structure of the equation, we can also provide an entirely new proof of Gibbons' conjecture in two dimensions. Moreover, we can prove a rigidity result for solutions with the same uniform limits at respective infinities. If time permits, we will sketch how our approach can be used to improve some results on one-dimensional symmetry of solutions in cylindrical domains, provided that the cylinder is convex.

Vlachos Theodoros (University of Ioannina)

Superconformal surfaces.

Superconformal surfaces in Euclidean space are the ones for which the ellipse of curvature at any point is a nondegenerate circle. They can be characterized as the surfaces for which a well-known pointwise inequality relating the intrinsic Gauß curvature with the extrinsic normal and mean curvatures, due to Wintgen and Guadalupe-Rodriguez for any codimension, reaches equality at all points. We show that any pedal surface to a 2-isotropic Euclidean surface is superconformal. Opposed to almost all known examples, superconformal surfaces in this class are not conformally equivalent to minimal surfaces. Furthermore, we discuss a duality phenomenon. A superconformal surface with arbitrary codimensionin flat Euclidean space has a (necessarily unique) dual superconformal surface if and only if the surface is S-Willmore. Duality means that both surfaces envelope the same central sphere congruence and are conformal with the induced metric. This is a joint work with Marcos Dajczer.